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Wireless Powered Communication Networks with Non-Ideal Circuit Power Consumption

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Abstract—Assuming non-ideal circuit power consumption at the energy harvesting (EH) nodes, we propose two practical protocols that optimize the performance of the harvest-then-transmit wireless powered communication networks (WPCNs) under two different objectives: (1) proportional fair (PF) resource allocation, and (2) sum rate maximization. These objectives lead to optimal allocations for the transmit power by the base station (BS), which broadcasts RF radiation over the downlink, and optimal durations of the EH phase and the uplink information transmission phases within the dynamic time-division multiple access (TDMA) frame. Compared to the *max-sum-rate* protocol, the *PF* protocol attains a higher level of system fairness at the expense of the sum rate degradation. The *PF* protocol is advantageous over the *max-sum-rate* protocol in terms of system fairness regardless of the circuit power consumption, whereas the uplink sum rates of both protocols converge when this power consumption increases.

Index Terms—energy harvesting, wireless powered communication networks, proportional fairness, non-ideal circuit power.

I. INTRODUCTION

The energy harvesting (EH) communication systems scavenge energy either from natural (such as, the solar or wind) [1], or man-made sources (such as, the radio-frequency (RF) radiation from power beacons) [2]. If the EH system relies on natural sources, it may be difficult to devise causal protocols that achieve sufficiently high rates, due to the intermittent nature of these energy sources. If an EH system is powered by RF radiation (e.g., a *wireless powered communication network* (WPCN)), it can even achieve rates comparable to that of conventional (non-EH) networks when intelligent policies for power and rate control are applied [3]. Typically, the WPCNs are optimized for their spectral efficiency, i.e., their sum rate over the uplink is maximized by dynamic adaptation of the base station (BS) transmit power and durations of the EH phase and the information transmission (IT) phase in each time-division multiple access (TDMA) frame [4]. However, despite its high spectral efficiency, a resource allocation strategy based upon the sum-rate maximization may unfairly distribute the system resources among the EH users (EHUs) at different distances from the BS. Instead, an opportunistic scheduling policy may facilitate system fairness, such as, the *proportional fair (PF) scheduling* [5]–[7]. Recently, [8] has proposed PF scheduling policies for the WPCNs, where a single EHU is scheduled for an IT in each TDMA frame. Clearly, resource sharing involving all EHUs in each TDMA frame would be much more efficient, and, therefore, in this paper, we focus on maximizing the uplink sum rate in a proportionally fair way.

Most of the studies assume that the EHUs spend the total amount of their harvested energy for RF transmissions, which significantly overestimate the predicted rates. However, the non-ideal electric circuitry of practical wireless transmitters consume significant additional power (e.g., AC/DC converter,

analog RF amplifier, and processor), which need to be taken into account [9]–[11]. In this paper, we develop two optimal protocols for the TDMA-based WPCNs with non-ideal circuit power consumption; The first one guarantees proportionally fair resource sharing among the EHUs, and the second one maximizes the sum rate over the uplink of the WPCN.

II. SYSTEM MODEL

We consider a WPCN with a single BS and K EHUs that operate in a wireless fading environment. Each network node is equipped with a single antenna. The network utilizes TDMA, and the time is divided into M TDMA frames of equal duration T (also referred to as the transmission epochs). Each epoch is divided into $K + 1$ phases, an EH phase and K successive IT phases, whose durations can be dynamically adjusted in each epoch. In epoch i , the duration of the EH phase, and the IT phase of k th EHU is $\tau_0(i)T$ and $\tau_k(i)T$, respectively, such that $\tau_0(i) + \tau_1(i) + \dots + \tau_K(i) = 1$. During the EH phase of epoch i , the BS broadcasts RF energy with an output power $p_0(i)$, which satisfies a maximum power constraint P_{max} (i.e., $0 \leq p_0(i) \leq P_{max}$), and an average power constraint P_{avg} (i.e., $E[p_0(i)\tau_0(i)] \leq P_{avg}$).

The random channel follows a quasi-static block fading model, where each fading block coincides with a single epoch. The channel between the BS and k th EHU is assumed reciprocal, and its fading power gain in epoch i is denoted by $x'_k(i)$. For convenience, we normalize these power gains as $x_k(i) = x'_k(i)/N_0$, where N_0 is the additive white Gaussian noise (AWGN) power. The average value of $x_k(i)$ is $\Omega_k = E[x'_k(i)]/N_0$, where $E[\cdot]$ denotes expectation. We assume perfect synchronization among the network nodes. The channel state information requirements are the following: the BS should know all K fading links, $\{x_k(i)\}_{k=1}^K$, whereas k th EHU should know only its own fading channel, $x_k(i)$.

A. Power Consumption Model for the Battery

The EHUs are equipped with rechargeable batteries that have low energy storage capacity and high discharge rate. Specifically, when an EHU transmits information, it completely spends all of the harvested energy in its battery during the previous BS broadcast in that same epoch, which is typically referred to as the *harvest-then-transmit* strategy [3], [4]. For the power consumption by the EHU, we apply a realistic model that incorporates both the transmit power and the non-ideal circuit power. Therefore, the total power consumed by the k th EHU in epoch i is given by [9]

$$p_{T,k}(i) = \begin{cases} P_k(i) + p_c, & P_k(i) > 0 \\ 0, & P_k(i) = 0 \end{cases} \quad (1)$$

where $P_k(i)$ is the transmit power of k th EHU, and p_c is the non-ideal circuit power consumed by that EHU during its IT phase. Note that p_c is fixed and independent of the EHU.

III. OPTIMAL RESOURCE ALLOCATION

In epoch i , the amount of harvested energy by k th EHU during the EH phase is given by

$$E_k(i) = \eta_k x_k(i) N_0 p_0(i) \tau_0(i) T, \quad (2)$$

where η_k is the conversion efficiency of k th EHU. Given the power consumption model (1), $P_k(i)$ is determined by

$$P_k(i) = \max \left\{ 0, \frac{E_k(i)}{\tau_k(i) T} - p_c \right\}, \quad (3)$$

where $E_k(i)$ is given by (2). Therefore, the achievable rate of k th EHU in epoch i is given by

$$r_k(i) = \tau_k(i) \log_2(1 + P_k(i) x_k(i)), \quad (4)$$

whereas the average achievable rate over M epochs is

$$\bar{R}_k = \lim_{M \rightarrow \infty} \frac{1}{M} \sum_{i=1}^M r_k(i). \quad (5)$$

A. Proportional fair resource allocation

The first protocol, referred to as the *PF* protocol, maximizes the sum of the logarithmic rates achieved by the network users, $\sum_{k=1}^K \log \bar{R}_k$ (c.f. [7], and references therein). Thus, when $M \rightarrow \infty$, we need to determine the optimal durations of the EH and IT phases and the optimal BS transmit power in each epoch by solving the following optimization problem:

$$\begin{aligned} & \text{Maximize} \quad \sum_{k=1}^K \log \left(\frac{1}{M} \sum_{i=1}^M \tau_k(i) \log_2(1 + x_k(i) P_k(i)) \right) \\ & \text{s.t.} \quad C1 : P_k(i) = \max \left\{ 0, \frac{E_k(i)}{\tau_k(i) T} - p_c \right\}, \forall i, 1 \leq k \leq K \\ & \quad C2 : \frac{1}{M} \sum_{i=1}^M p_0(i) \tau_0(i) \leq P_{avg} \\ & \quad C3 : 0 \leq p_0(i) \leq P_{max}, \forall i \\ & \quad C4 : \sum_{k=1}^K \tau_k(i) = 1 - \tau_0(i), \forall i \\ & \quad C5 : 0 < \tau_k(i) < 1, \forall i, 0 \leq k \leq K. \end{aligned} \quad (6)$$

The solution of (6) is given by the following theorem.

Theorem 1: The optimal transmit power of the BS is

$$p_0^*(i) = \begin{cases} P_{max}, & \sum_{k=1}^K \frac{1}{\bar{R}_k} \frac{a_k(i)}{1 - p_c x_k(i) + z_k(i)} > \lambda \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

The optimal durations of EH and IT phases are determined by

$$\tau_0^*(i) = \frac{1}{1 + \sum_{k=1}^K \frac{a_k(i) P_{max}}{z_k(i)}}, \quad (8)$$

$$\tau_k^*(i) = \frac{a_k(i) P_{max}}{z_k(i)} \tau_0^*(i), \quad 1 \leq k \leq K, \quad (9)$$

respectively, where $a_k(i) = \eta_k N_0 x_k^2(i)$. In (7), (8) and (9), $z_k(i)$ are auxiliary variables that are determined by

$$\begin{aligned} z_k(i) &= -(1 - p_c x_k(i)) \\ &\times \left[1 + \frac{1}{W(-(1 - p_c x_k(i)) e^{-1 - \beta_i P_{max} \bar{R}_k})} \right], \end{aligned} \quad (10)$$

where $W(\cdot)$ is Lambert- W function, and β_i is found as the root of the following transcendental equation,

$$\begin{aligned} & \sum_{k=1}^K \frac{1}{\bar{R}_k} \frac{a_k(i)}{1 - p_c x_k(i)} W \left(-(1 - p_c x_k(i)) e^{-1 - \beta_i P_{max} \bar{R}_k} \right) \\ & + \beta_i + \lambda = 0 \end{aligned} \quad (11)$$

The constant λ is determined from C2 in (6) set to equality.

Proof: Please refer to Appendix A. ■

In practice, the values of λ and \bar{R}_k may not be available in advance. For an online estimation of λ , we apply the stochastic gradient descent method [12], as

$$\hat{\lambda}(i) = \hat{\lambda}(i-1) + \gamma_0 \left(\frac{1}{i-1} \sum_{n=1}^{i-1} p_0(n) \tau_0(n) - P_{avg} \right), \quad (12)$$

where γ_0 is some small step size. The rate \bar{R}_k can be also updated online according to a simple iterative rule,

$$\hat{R}_k(i) = \frac{i-1}{i} \hat{R}_k(i-1) + \frac{1}{i} r_k(i). \quad (13)$$

Actually, each iteration of $\hat{R}_k(i)$ is based upon an ever increasing window size, equal to the elapsed session time i , which guarantees the maximization of $\sum_{k=1}^K \log \bar{R}_k$ [5, Lemma 4]. The practical implementation of the proposed policy at the BS is outlined by Algorithm 1.

Algorithm 1 PF protocol implementation at the BS

```

1: Initialize  $\hat{\lambda}, \hat{R}_k \forall k$ . Set  $\gamma_0$  and  $i = 1$ ;
2: repeat
3:   Determine  $b(i) = \sum_{k=1}^K \frac{1}{\bar{R}_k} \frac{a_k(i)}{1 - p_c x_k(i) + z_k(i)}$ ;
4:   if  $b(i) > \hat{\lambda}$  then
5:     Calculate  $\tau_0(i)$  from (8);
6:     Calculate  $\tau_k(i)$  from (9), and feedback to EHUs;
7:     Calculate  $r_k(i)$  from (4), and feedback to EHUs;
8:     Broadcast RF energy at  $p_0(i) = P_{max}$  for  $\tau_0(i)T$ ;
9:   else
10:    Set  $p_0(i) = 0, \tau_k(i) = 0$  and  $r_k = 0, \forall k$ ;
11:     $\hat{\lambda} \leftarrow \hat{\lambda} + \gamma_0 \left( \frac{1}{i} \sum_{n=1}^i p_0(n) \tau_0(n) - P_{avg} \right)$ ;
12:     $\hat{R}_k \leftarrow \frac{i-1}{i} \hat{R}_k + \frac{1}{i} r_k(i), \forall k$ ;
13:     $i \leftarrow i + 1$ .
14: until  $i \leq M$ 

```

B. Sum rate maximization

The second protocol, referred to as the *max-sum-rate* protocol, aims at maximizing the achievable average rate in the uplink of the WPCN, $\sum_{k=1}^K \bar{R}_k$. For such an objective, the relevant optimization problem is given by

$$\begin{aligned} & \text{Maximize} \quad \frac{1}{M} \sum_{i=1}^M \sum_{k=1}^K \tau_k(i) \log_2(1 + x_k(i) P_k(i)) \\ & \text{s.t.} \quad C1 : P_k(i) = \max \left\{ 0, \frac{E_k(i)}{\tau_k(i) T} - p_c \right\}, \forall i, 1 \leq k \leq K \\ & \quad C2 : \frac{1}{M} \sum_{i=1}^M p_0(i) \tau_0(i) \leq P_{avg} \\ & \quad C3 : 0 \leq p_0(i) \leq P_{max}, \forall i \\ & \quad C4 : \sum_{k=1}^K \tau_k(i) = 1 - \tau_0(i), \forall i \\ & \quad C5 : 0 < \tau_k(i) < 1, \forall i, 0 \leq k \leq K. \end{aligned} \quad (14)$$

The solution of (14) is a corollary of Theorem 1.

Corollary 1: The optimal allocations for the BS transmit power, $p_0^*(i)$, the duration of the EH phase, $\tau_0^*(i)$, and the duration of the IT phases, $\tau_k^*(i)$, are given by Theorem 1, where $\bar{R}_1 = \bar{R}_2 = \dots = \bar{R}_K = 1$.

Proof: Similar to the proof of Theorem 1 (a.k.a. Appendix A), we firstly assume that the constraint $C1$ is strictly positive, which is validated by the optimal solution. In this case, introducing the change of variables $e(i) = p_0(i)\tau_i(i)$, transforms (14) into a convex optimization problem. Its Lagrangian corresponds to (17) with the log operator omitted (that yields the sum-rate maximization, instead of the sum-log-rate maximization). As a result, the Lagrangian derivatives with respect to $e(i)$, $\tau_k(i)$, and $\tau_0(i)$ correspond to (18), (19), and (20), respectively, with \bar{R}_k set to unity. ■

The practical implementation of Corollary 1 is similar to Algorithm 1, with $\bar{R}_k = 1, \forall k$ (and step 12 removed).

Note that, when $p_c = 0$, (14) reduces to [4, Eq. (3)], and its solution is given by [4, Theorem 1]. Additionally, the solution of (14) in the special case of $K = 1$ is given by [4, Theorem 2], but the presented approach to its proof is not applicable to the case of arbitrary K . Corollary 1 presents the general solution of the sum rate maximization problem for arbitrary values of p_c and K .

IV. NUMERICAL RESULTS

For the numerical examples, we assume a Rayleigh fading environment, with the path-loss exponent $\alpha = 3$ and the path loss of 30dB at reference distance of 1m, i.e., $E[x_k'(i)] = 10^{-3} D_k^{-\alpha}$. Five EHUs are placed at different distances from the BS: $D_1 = 10\text{m}$, $D_2 = 12.5\text{m}$, $D_3 = 15\text{m}$, $D_4 = 17\text{m}$, and $D_5 = 18.8\text{m}$. We also set $P_{max} = 5P_{avg}$, and $N_0 = 10^{-12}\text{W}$. As the measure for the system fairness, we adopt the Jain's fairness index [14], $J = (\sum_{k=1}^K \bar{R}_k)^2 / (K \sum_{k=1}^K \bar{R}_k^2)$.

Fig. 1 considers the effect of the processing cost at $P_{avg} = 1\text{W}$. Fig. 1a depicts the sum rate over the uplink of the WPCN, $(1/M) \sum_{k=1}^K \sum_{i=1}^M r_k(i)$, whereas Fig. 1b depicts the system fairness, J . As p_c increases, the sum rates steadily decrease, while the fairness index is kept nearly constant for both proposed protocols. For a given K , the sum rate difference and the fairness index difference between the two proposed protocols are nearly independent of p_c (i.e., the all the curves are nearly parallel). As K increases, the *max-sum-rate* protocol exerts its increasing advantage over the *PF* protocol in terms of the sum rate. On the other hand, relative to the *max-sum-rate* protocol, the *PF* protocol attains a higher level of system fairness for all K , and this advantage increases with K .

Fig. 2 depicts the uplink sum rate vs. P_{avg} for various p_c as parameter, by assuming that all EHUs are at the same distance from BS (i.e., $D_k = 10\text{m}$, $\forall k$, and, therefore, $J = 1$). The two proposed protocols are compared against the benchmark protocol studied in [3], which maximizes the uplink sum rate by fixing the output power at the BS (i.e., $p_0(i) = P_0 = \text{const.}, \forall i$), and adjusting only the durations of the EH/IT phases, $\tau_k(i)$, as per [3, Eq. (10)]. Note, the benchmark only applies to the case of $p_c = 0$, but not $p_c > 0$. For a fair comparison, P_0 is selected in order to satisfy our average power constraint $C2$, i.e., $E[P_0\tau_0(i)] = P_{avg}$. Fig. 2 indicates that the advantage of the *max-sum-rate* protocol in

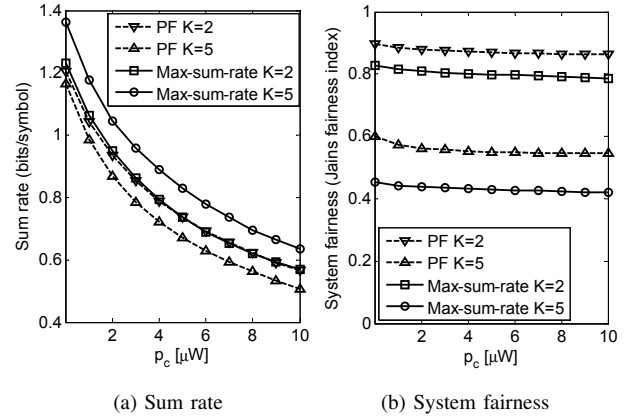


Fig. 1: Tradeoff between the sum rate and fairness

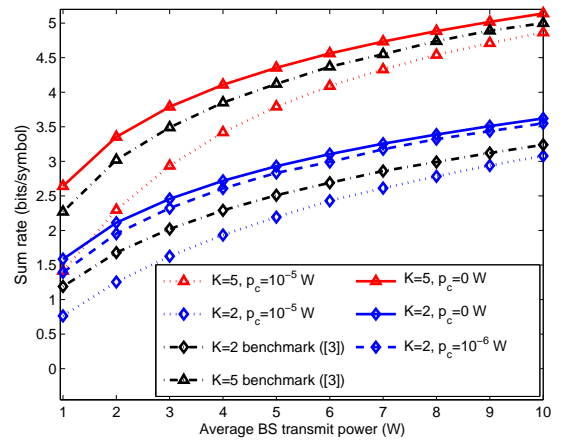


Fig. 2: The effect of BS average transmit power on sum rate

terms of the sum rate is less pronounced with increasing P_{avg} , especially for larger values of K .

V. CONCLUSION

In this paper, we have proposed and studied two protocol schemes for the WPCNs, one that guarantees proportionally fair resource allocation among EHUs at different distances from the BS, and another one that guarantees maximization of the sum rate over the uplink. The optimal power and time allocations of both protocols account for the non-ideal circuit power consumption at the EHUs, which facilitates the practicality of these protocols.

APPENDIX A

PROOF OF THEOREM 1

Let us assume the optimal solution of (6) is such that the constraint $C1$ is strictly positive, i.e.,

$$P_k(i) = \frac{\eta_k x_k(i) N_0 p_0(i) \tau_0(i)}{\tau_k(i)} - p_c > 0, \quad \forall i, \forall k. \quad (15)$$

In this case, after applying (15) and the variables change $e(i) = p_0(i)\tau_0(i)$, (6) is transformed into

$$\begin{aligned} \text{Maximize}_{\tau_k(i), \tau_0(i), e(i)} \quad & \sum_{k=1}^K \log \left(\frac{1}{M} \sum_{i=1}^M \tau_k(i) \right) \\ & \times \log_2 \left(1 - x_k(i)p_c + \frac{a_k(i)e(i)}{\tau_k(i)} \right) \end{aligned}$$

$$\begin{aligned}
\text{s.t.} \quad & \bar{C}2 : \frac{1}{M} \sum_{i=1}^M e(i) \leq P_{avg} \\
& \bar{C}3 : 0 \leq e(i) \leq P_{max} \tau_0(i), \forall i \\
& \bar{C}4 : \sum_{k=1}^K \tau_k(i) = 1 - \tau_0(i), \forall i \\
& \bar{C}5 : 0 < \tau_n(i) < 1, \forall i, 0 \leq n \leq K.
\end{aligned} \quad (16)$$

The function $\tau_k(i) \log_2(1 - p_c x_k(i) + a_k(i)e(i)/\tau_k(i))$ is the perspective of $\log_2(1 - p_c x_k(i) + a_k(i)e(i))$, and, therefore, it is jointly concave in $e(i)$ and $\tau_k(i)$ [13, Section 3.2.6]. The inner sum of the objective function of (16) is positive and concave in $e(i)$ and $\{\tau_k(i)\}_{k=1}^K$, and, therefore, its logarithm is also concave [13, Section 3.5.1]. The constraints are all affine (i.e., convex) functions. As a result, (16) is a convex optimization problem, which can be solved by applying the Lagrangian dual method. Its Lagrangian is given by

$$\begin{aligned}
\mathcal{L} = & \sum_{k=1}^K \log \left(\frac{1}{M} \sum_{i=1}^M \tau_k(i) \log_2 \left(1 - x_k(i)p_c + \frac{a_k(i)e(i)}{\tau_k(i)} \right) \right) \\
& - \lambda \left(\frac{1}{M} \sum_{i=1}^M e(i) - P_{avg} \right) - \sum_{i=1}^M \beta_i (e(i) - P_{max} \tau_0(i)) \\
& + \sum_{i=1}^M \alpha_i e(i) - \sum_{i=1}^M \mu_i \left(\tau_0(i) + \sum_{k=1}^K \tau_k(i) - 1 \right)
\end{aligned} \quad (17)$$

where the Lagrange multipliers λ , α_i , β_i , and μ_i are associated with $\bar{C}2$, the left hand side of $\bar{C}3$, the right hand side of $\bar{C}3$, and $\bar{C}4$, respectively. They satisfy the following slackness conditions: $\lambda(\sum_{i=1}^M e(i)/M - P_{avg}) = \alpha_i e(i) = \beta_i (e(i) - P_{max} \tau_0(i)) = \mu_i (\tau_0(i) + \sum_{k=1}^K \tau_k(i) - 1) = 0$. The derivatives of (17) with respect to $e(i)$, $\tau_k(i)$, and $\tau_0(i)$, set equal to zero, yield the following equation set:

$$\sum_{k=1}^K \frac{1}{\bar{R}_k} \frac{a_k(i)}{1 - p_c x_k(i) + \frac{a_k(i)e(i)}{\tau_k(i)}} - \lambda + \alpha_i - \beta_i = 0 \quad (18)$$

$$\log \left(1 - x_k(i)p_c + \frac{a_k(i)e(i)}{\tau_k(i)} \right) - \frac{\frac{a_k(i)e(i)}{\tau_k(i)}}{1 - x_k(i)p_c + \frac{a_k(i)e(i)}{\tau_k(i)}} = \bar{R}_k \mu_i \quad (19)$$

$$\beta_i P_{max} - \mu_i = 0. \quad (20)$$

Case 1: Let $e(i) = 0$. Then, no power is allocated to epoch i , yielding $p_0^*(i) = 0$.

Case 2: Let $e(i) = P_{max} \tau_0(i)$. Due to the slackness conditions, we obtain $\alpha_i = 0$, $\beta_i > 0$, and $\mu_i > 0$. Thus, (19) reduces to

$$\log(1 - x_k(i)p_c + z_k(i)) - \frac{z_k(i)}{1 - x_k(i)p_c + z_k(i)} = \beta_i P_{max} \bar{R}_k, \quad (21)$$

where

$$z_k(i) = \frac{a_k(i) P_{max} \tau_0(i)}{\tau_k(i)}. \quad (22)$$

The closed form solution of (21) is given by (10). Eq. (18) yields

$$\beta_i = \sum_{k=1}^K \frac{1}{\bar{R}_k} \frac{a_k(i)}{1 - p_c x_k(i) + z_k(i)} - \lambda > 0. \quad (23)$$

The combination of (23) and (10) yields (11). Additionally, (23) leads to the condition for $p_0(i) = P_{max}$ in (7).

In order to verify the assumption (15), we notice that, since $\beta_i > 0$, the left hand side of (21) should be positive, i.e.,

$$\log(1 - x_k(i)p_c + z_k(i)) - \frac{z_k(i)}{1 - x_k(i)p_c + z_k(i)} > 0. \quad (24)$$

For arbitrary $x_k(i)p_c > 0$, (24) is satisfied iff $z_k(i) - x_k(i)p_c > 0$, which is equivalent to the assumption (15).

Case 3: Let $0 < p_0(i) < P_{max}$. Due to the slackness conditions, $\alpha_i = \beta_i = 0$. In this case, (18) and (19) respectively reduce to

$$\sum_{k=1}^K \frac{1}{\bar{R}_k} \frac{a_k(i)}{1 - p_c x_k(i) + z_k(i)} = \lambda, \quad (25)$$

$$z_k(i) = -(1 - p_c x_k(i)) \left[1 + \frac{1}{W(-(1 - p_c x_k(i)) e^{-1})} \right]. \quad (26)$$

By introducing (26) into (25), we obtain an expression which can be satisfied only for one specific set of values for $\{x_k(i)\}_{k=1}^K$. Since the channel gains are assumed to follow continuous PDFs, occurrence probability of this case is zero.

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